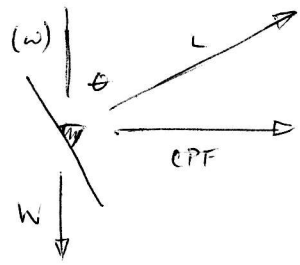

where L = lift, CPF = centripetal force, W = weight
θ = AoB

$$\tan\theta = \frac{CPF}{W} = \frac{mv^2/r}{W}$$

but $m = W/g$

so $\tan\theta = \frac{V^2}{gr}$ $\quad\therefore\quad r = \frac{V^2}{g\tan\theta}$

which is great but that's all in SI units so to convert to a simplified version using Mach No and calling it nm/min

radius = $(nm/min)^2 \times 0.103$ at 30° AoB

Of course I might have all the conversion factors wrong but...

Extending the thinking process for minimum radius turns:

$V_m = V_0\sqrt{n}$ where V_m = V manoeuvre stall, V_0 = basic stall & n = load factor

$$n = \frac{L}{W} = \frac{1}{\cos\theta}$$

$$V_m^2 = V_0^2\frac{1}{\cos\theta} \qquad \text{so} \qquad r = \frac{V_0^2}{g\tan\theta}\times\frac{1}{\cos\theta} = \frac{V_0^2}{g\sin\theta}$$

At stalling speed $L = W = C_{L\max}\,{}^1\!/_2\,\rho V_0^2 S$

$$\text{so } V_0^2 = \frac{2W}{C_{L\max}\rho S} \qquad \text{thus } r = \left(\frac{1}{C_{L\max}\sin\theta}\right)\frac{W}{S}\frac{1}{\rho}\frac{2}{g}$$

so for min radius, term 1 must be a minimum. $\frac{1}{\sin\theta}$ trends to 1 as θ to 90°. Therefore increase speed to $C_{L\max}$, W/S measures wing loading and must be a minimum and ρ density must be maximum so sea level.

And finally maximum rate turns

$$\sin\theta = \frac{CPF}{L} = \frac{W}{S}\frac{V^2}{r}\frac{1}{L}$$

$$\text{rate of turn} = \frac{V}{r} = \frac{L\sin\theta\, g}{WV} \qquad \text{but } L = C_L\,{}^1\!/_2\,\rho V^2 S$$

$$= \frac{C_L\,{}^1\!/_2\,\rho V^2 S\sin\theta\, g}{WV}$$

$$= C_L V\sin\theta\,\rho\left(\frac{S}{W}\right)\frac{g}{2}$$

so for a max rate turn, $C_L V\sin\theta$ must be a max. As before $\sin\theta$ trends to 1 as θ to 90°. Therefore increase bank requires increased speed as long as C_L does not decrease faster than $V\sin\theta$ increases. Wing loading must be a minimum and ρ density must be maximum at sea level.

QED.